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Welded Continuous Frames and Their Components

MOMENT-CURVATURE-THRUST PROGRAM FOR WIDE-FLANGE SHAPES

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by
Yuhshi Fukumoto

Fritz Engineering Laboratory Report No. 205A.37

WELDED CONTINUOUS FRAMES AND THEIR COMPONENTS

MOMENT-CURVATURE-THRUST PROGRAM

FOR WIDE-FLANGE SHAPES

by

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SYNOPSIS

A computational procedure for the determination of the moment-curvature-thrust relationships is presented for as-rolled steel wide-flange beam-columns bent about their strong axis.

The computer program (WIZ language, GE 225 computer) is developed for the determination of the moment-curvature-thrust curves of wide-flange shapes under the presence of the cooling residual stresses.

Comparisons of the moment-curvature-thrust curves are given for different wide-flange shapes, yield stresses and residual stress distributions.

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1. INTRODUCTION

The determination of the M- ϕ -P relationships is accomplished by assuming a specific stress distribution, and thus a yielded pattern, and then computing the corresponding value of P, M and ϕ from geometry and equilibrium, that is,

$$P = \int_A \sigma dA ; \quad M = \int_A \sigma y dA ; \quad \phi = \frac{\epsilon_1 - \epsilon_2}{d}$$

The stress-strain diagram of the member is assumed to be ideally elastic-plastic (Fig. 1), and the cross section is an idealized wide-flange shape where the variation in the thickness of the flanges and the fillets at the toe are neglected (Fig. 2). Cooling residual stresses are present along the member with the assumed distribution⁽¹⁾ as shown in Fig. 2.

As the moment is increased under a given constant axial thrust, yielding will first occur at the outside tips of the compression flange where the compressive residual stress is maximum, and as M is increased, yielding will continue to penetrate through the flange. Eventually yielding occurs in the tension flange and the web, and finally the full plastic condition is developed.

The non-dimensionalized $M/M_y - \phi/\phi_y - P/P_y$ relationships about the strong axis have been determined for the following five different stages of yielding in wide-flange sections containing residual stresses:

- (1) Elastic case (Fig. 3a)
- (2) Partial yielding in the compression flange, with yielding progressing from the flange tips towards the center while the web and the tension flange remain elastic (Fig. 3b).
- (3) Partial yielding in the compression flange, in the tension zones of the web and in the tension flange (Fig. 3c).
- (4) Partial yielding in the compressed part of the web, while the remainder of the web and the tension flange are elastic and the compression flange is fully plastic (Fig. 3d).
- (5) Partial yielding in both the compression and tension zones of the web, and full plasticity in the compression and tension flanges (Fig. 3e).

The five yielded patterns enumerated above do not include all the stages of yielding which are encountered in a wide-flange shape which contains the residual stresses shown

in Fig. 2, but they permit the construction of the M- ϕ -P curves over the ranges of most importance.

The equations for M- ϕ -P relationships are quite complicated and cumbersome, and a semi-graphical method has been used previously to determine M- ϕ -P curves for specified cross sections. (1) (2)

Since it was desired to utilize a digital computer for the work, the equations here were solved analytically. The formulas are summarized in Table 1 in Refs. (3) and (4). The table contains the following items for each different yielding pattern shown in Fig. 3.

- (1) Given parameters (that is, cross-sectional dimensions, P/P_y , ϕ/ϕ_y , material properties).
- (2) Limits of the formulas.
- (3) The extent at yielding, α , γ , etc.
- (4) The moment equations which correspond to the specified curvature, thrust and the yield pattern.

2. COMPUTER PROGRAM

A computer program (WIZ language) for the digital computer GE 225 has been set up for the determination of the M- ϕ -P relationships. The program includes the M- ϕ -P relationships for cases (b), (d) and (e) in Fig. 3.*

A general flow diagram of the M- ϕ -P program is shown in Fig. 4. The computational procedures are explained as follows:

(1) The information required as input data:*

(a) Cross-sectional dimensions, that is,

b, d, t, and w (see Fig. 2).

(b) $R_c = \sigma_{rc}/\sigma_y$, ratio of the maximum compressive residual stresses, σ_{rc} , to the yield stress, σ_y .

(c) Increments of curvature, k_1 , k_2 , k_3 and k_4 , where k_1 is the increment of curvature for the yield pattern of Fig. 3 (b), k_2 is for case (d) in Fig. 3 or $\phi/\phi_y=5.0$, k_3 is for case (e) in Fig. 3 or $\phi/\phi_y=10.0$, and k_4 is for $\phi/\phi_y=20$.

* The program for case (c) in Fig. 3 is developed only for $P/P_y=0$ as a separate program. Input instructions are given in Appendix B for this case and Appendix A for the general case.

- (d) The value of P/P_y . The moment-curvature relationships will be computed under a constant value of P/P_y .
- (2) The program will give the shape factor, f , and also the non-dimensionalized modified plastic moment, M_{pc}/M_y , for the specified wide-flange shape and the axial thrust.
- (3) The program calculates the yield extensions α , γ , or γ_1 , and γ_2 for the specified ϕ/ϕ_y . The computer starts to calculate with the initial value of ϕ/ϕ_y which corresponds to the elastic limit. It will check the calculated yield extensions with the limits which are defined for each different yield pattern, and it will determine the correct yield pattern.
- (4) The computer will calculate the value M/M_y for the ϕ/ϕ_y and for the corresponding yield pattern. If the specified ϕ/ϕ_y is beyond or between these yield patterns, the program will give new values of ϕ/ϕ_y until it finds the corresponding yield patterns.

- (5) The $M/M_y - \phi/\phi_y$ yield extensions are printed as the results and at the same time the $M/M_y - \phi/\phi_y$ values are punched on cards.*
- (6) If the ϕ/ϕ_y is less than 20,** the program will repeat the same process for the new ϕ/ϕ_y . The increments of ϕ/ϕ_y will be given by Step (1) (c). The program will return to "start" when ϕ/ϕ_y reaches 20 and read another set of input data.

* These cards can be used as the input data for calculating the column deflection curves or the deformations of the beam-columns in the inelastic range.

** $\phi/\phi_y=20$ is arbitrarily picked to be assumed the value which gives M/M_y in the nearly flat portion of the curve.

3. DISCUSSION

Comparisons of the M- ϕ -P relationships for different wide-flange shapes and for different magnitudes of the yield stresses and the residual stresses will be made in this section.

Wide-Flange Shapes

The M- ϕ -P relationships can be presented as a family of curves, with M/M_y as the ordinate and ϕ/ϕ_y as the abscissa; each curve is for a constant value of P/P_y . Such curves for the 8WF31 section are shown in Fig. 5. Also shown on the curves in Fig. 5 are the zones in which the various patterns of yielding given in Fig. 3 occur. It can be seen that yield patterns (b) and (c) are the most prevalent ones for $P/P_y=0$ and yield patterns (b) and (d) are the most prevalent ones if an appreciable axial thrust exists ($P/P_y \geq 0.2$).

In Fig. 6 the M- ϕ -P curves are compared for different wide-flange shapes. One is for the 8WF31 section ($f = 1.107$) and the other is for the 14WF246 section ($f = 1.167$) for $P/P_y = 0.4$. These two shapes represent the lower and higher shape factor among the common wide flange shapes.*

* For wide-flange shapes normally used as columns the shape factor varies from 1.10 to 1.23 with an average value of 1.137 and a mode (most frequently observed value) of 1.115(5).

In Fig. 7 the curves for the 8WF31 and the 14WF246 section are shown with M/M_y as ordinate. The $M-\phi-P$ curves become close together and are nearly independent of the shape factors when the curves are presented on the non-dimensionalized $M/M_p - \phi/\phi_y$ ordinates for a constant value of P/P_y .

Yield Stresses

When the $M-\phi-P$ relationships are given by the non-dimensionalized $M/M_y - \phi/\phi_y - P/P_y$ parameters, the relationships are independent of the influence of the different yield stress levels.

Residual Stresses

In Fig. 8 the $M-\phi-P$ curves are shown with different magnitudes of the residual stresses, $R_c = \sigma_{rc}/\sigma_y$, for the 8WF31 section and $P/P_y = 0.2$.

The different magnitudes of the residual stresses change the $M-\phi-P$ curves considerably at the early stage (at the commencement of yielding) where the inelastic lateral instability phenomena become important.⁽³⁾⁽⁴⁾

4. ACKNOWLEDGEMENTS

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The author wishes to express his appreciation to Dr. Theodore V. Galambos for his fruitful suggestions and to Messrs. Pierre Chevin and Peter Adams for their help in programming of the computations.

5. FIGURES

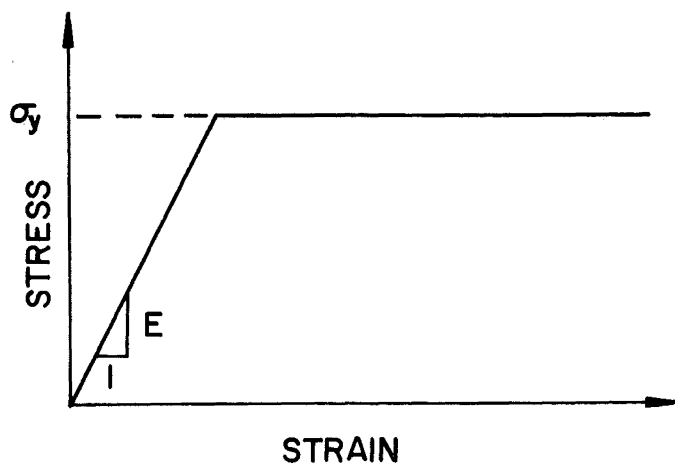


FIG. 1 IDEALIZED STRESS-STRAIN DIAGRAM

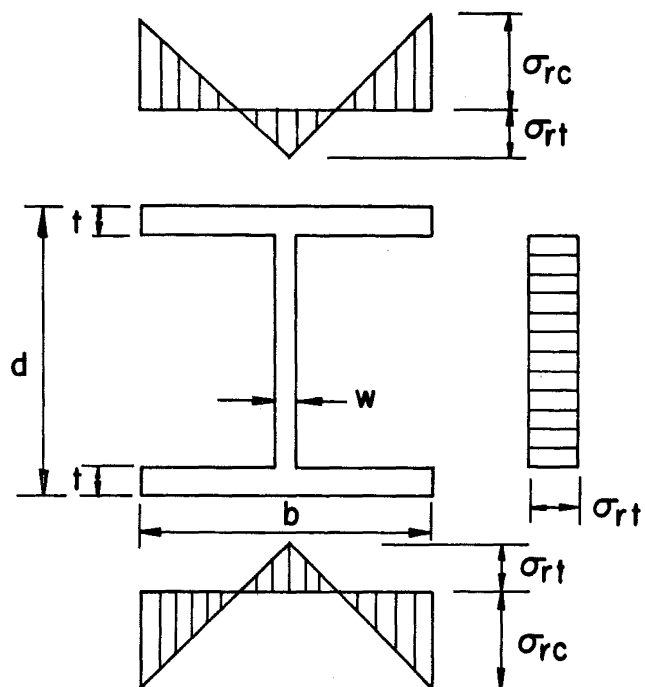


FIG. 2 ASSUMED COOLING RESIDUAL STRESS PATTERN

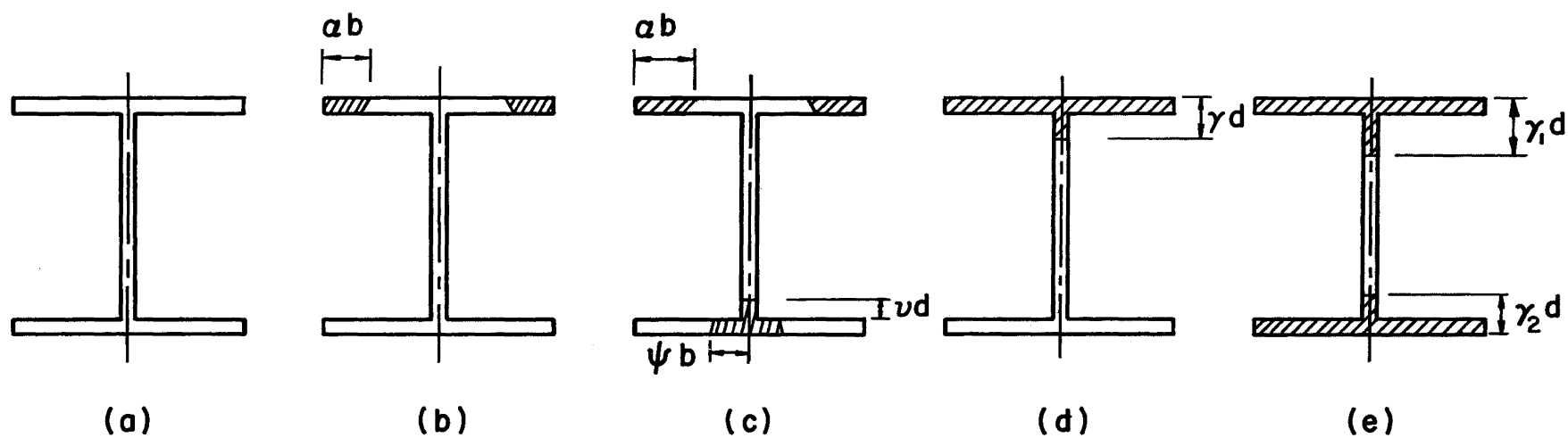


FIG. 3 YIELD PATTERNS FOR WIDE-FLANGE CROSS-SECTION

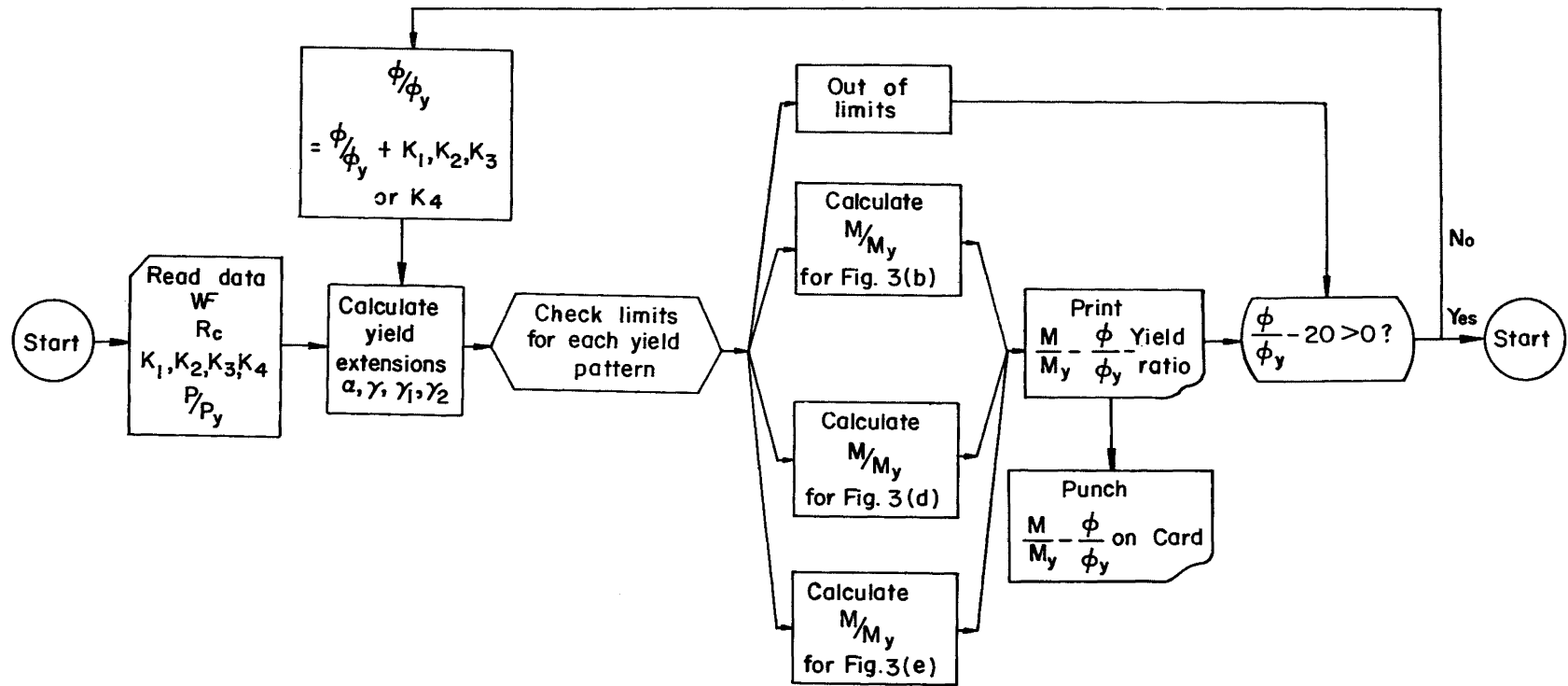


FIG. 4 BLOCK DIAGRAM FOR COMPUTATIONAL PROCEDURES

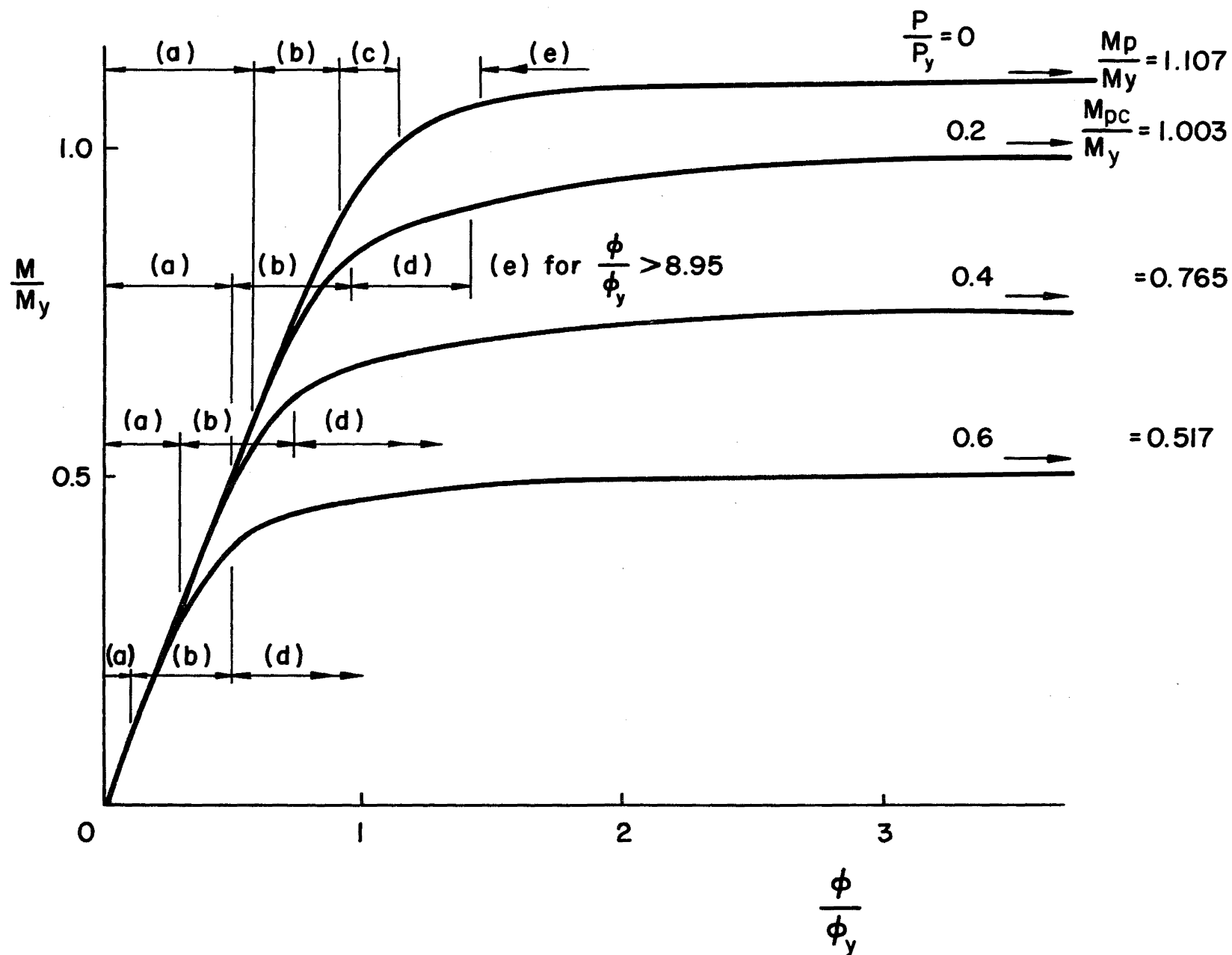


FIG. 5 MOMENT-CURVATURE-THRUST RELATIONSHIPS,
FOR STRONG AXIS BENDING, 8WF31, $R_c=0.3$

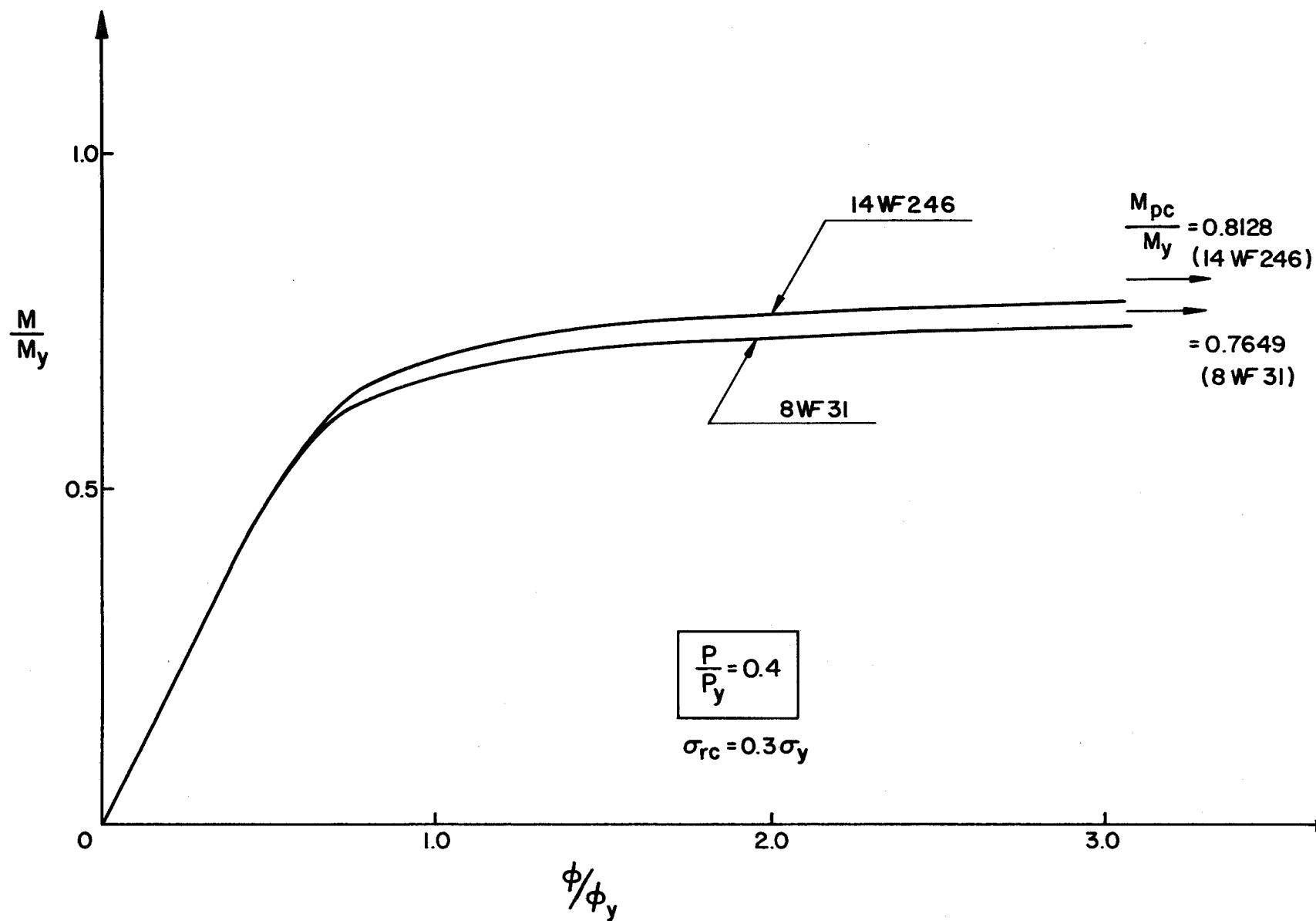


FIG. 6 MOMENT-CURVATURE-THRUST RELATIONSHIPS
FOR STRONG AXIS BENDING, 8WF31, 14WF246

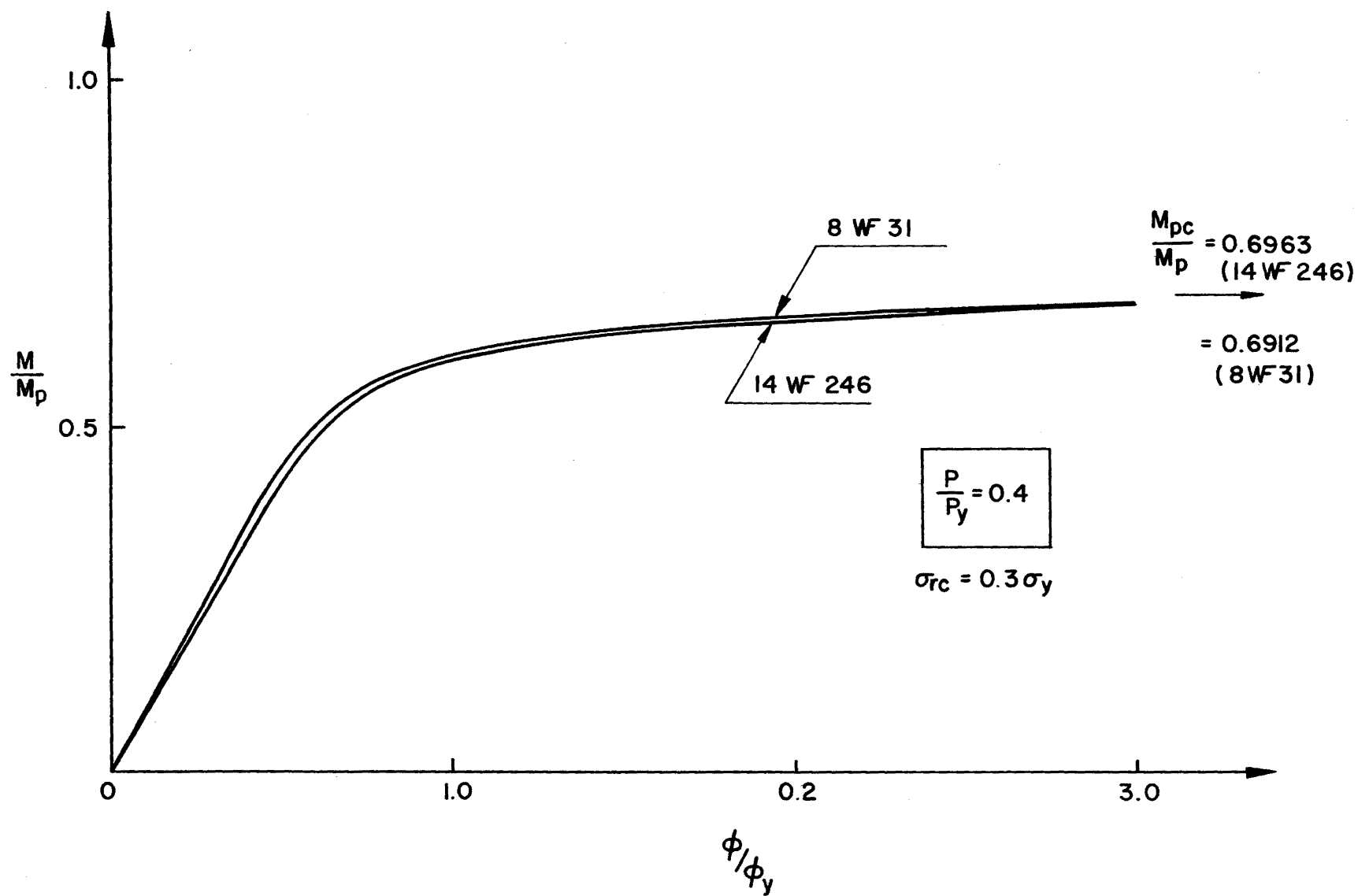


FIG. 7 MOMENT-CURVATURE-THRUST RELATIONSHIPS,
USING M/M_p AS ORDINATE

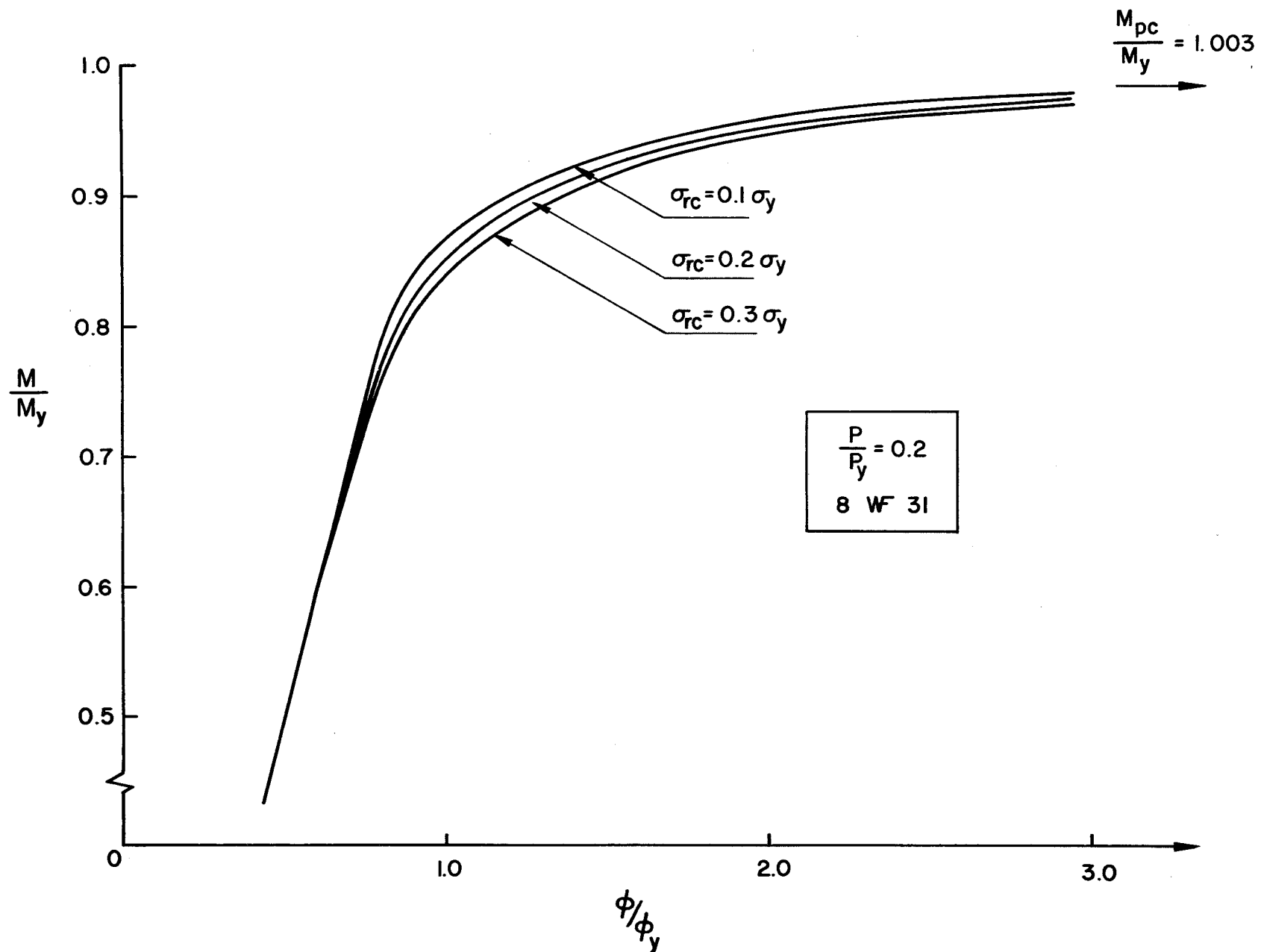


FIG. 8 MOMENT-CURVATURE-THRUST RELATIONSHIPS,
SHOWING INFLUENCE OF RESIDUAL STRESS
LEVEL

6. APPENDICES

A. INSTRUCTION FOR THE M- ϕ -P PROGRAM

Input Data

Punch the corresponding numerical data appearing in the following items:

1st card (a): WF B D T W RC

Example 8WF31, $\nabla_{rc} = 0.3\nabla_y$

Punch 80*31 8.0 8.0 0.433 0.288 0.3

2nd card (b): k_1 k_2 k_3 k_4

Example 0.05 0.2 1 2

3rd card (c): P/ P_y

If only one set of M/M_y versus ϕ/ϕ_y is desired, after (c), place the "END" card followed by a blank card. The program will stop by reading the End card as a data.

If other sets of M/M_y versus ϕ/ϕ_y are needed, the following different cases may be incorporated:

(1) Only P/P_y changes; after (c) place another card (c) with the new P/P_y .

(2) Either one of the coefficients k_1 , k_2 , k_3 , k_4 changes, after (c) place a card with "-1", then

* 0 is zero not the letter.

card (b) with the new set of k_1, k_2, k_3, k_4 ,
and card (c) with the new P/P_y .

- (3) Either one of the values WF shape on R_c changes;
after (c) place a card with "-1-1", then cards
(a) (b) and (c) with the new set of values.

In the data deck, the first card will be of the
card (a) type and the last one will be of the card (c) type,
then "END" card and a blank card.

Computer Output

The following is an example of the computer output:

Printed

140246 \$ WF 1.1672498 + 00 \$ F
(Interpreted as 14WF246) (f = 1.1672498; shape factor)

2.0000000 - 01 \$ POPY 1.0508321 + 00 \$ MPCOMY
($P/P_y = 0.2$) ($M_{pc}/M_y = 1.0508321$)

M/M_y	$0/0_y$	AL	GA	GA1	GA2
	(ϕ/ϕ_y)	(α)	(γ)	(γ_1)	(γ_2)

Punched

One pair of M/M_y versus ϕ/ϕ_y values on each
card. Interpretation of results:

- (1) in the last two printed lines, that is, for

$\phi/\phi_y = 200$ and $\phi/\phi_y = -1$, the value of γ, γ_1 ,

and γ_2 are not to be considered. The computer simply prints what is in the corresponding memory location.

$$(2) \text{ For } \phi/\phi_y = 200 (= \infty) \quad M/M_y = M_{pc}/M_y$$

$$\phi/\phi_y = -1 \quad M/M_y = -1$$

These four values are placed at the end of the output data artificially.

B. INSTRUCTION FOR THE M- ϕ -P PROGRAM FOR CASE (c) IN FIG. 3. ($P/P_y = 0$)

Input Data

Punch the corresponding numerical data appearing in the following items:

1st card (a): B D T W RC

2nd card (b): ϕ/ϕ_y (=F)

The elastic limit value of ϕ/ϕ_y may be punched on the 2nd card.

If only one set of M/M_y versus ϕ/ϕ_y is desired, after (b), place the "END" card followed by a blank card. The program will stop by reading the End card as a data.

If other sets of M/M_y versus ϕ/ϕ_y are needed, after (b),

place a card (a) with new set of B D T W RC and card (b) with the new ϕ/ϕ_y .

In the data deck, the first card will be of the card (a) type and the last one will be of the card (b) type, then "END" card and a blank card.

Computer Output

The following is an example of the computer output:

Printed

A1

2.4327348-01
($\alpha_1=0.24327348$)*

O/OY	MO/MY	ALPHA	H	G
(ϕ/ϕ_y)	(M/M _y)	(α)	(ψ)	(ν)

* α_1 (=A1) represents the lower limit of α at which yielding starts to penetrate into the tension zones of the web and the tension flange.

7. NOTATION

A	Area of cross section
b	Flange width
d	Depth of section
f	Shape factor, $f = Z/S$
M	Bending moment
M_p	Plastic moment
M_{pc}	Plastic moment modified to include the effect of axial thrust
M_y	Moment at which yielding first occurs in flexure, $M_y = S\sigma_y$
P	Axial thrust
P_y	Axial thrust corresponding to yield stress level, $P_y = A\sigma_y$
R_c, R_t	$R_c = \sigma_{rc}/\sigma_y$, $R_t = \sigma_{rt}/\sigma_y$
S	Section modulus about strong axis
t	Flange thickness
w	Web thickness
Z	Plastic modulus
α, γ_1 γ_2, γ_3	Coefficients indicating yielding of cross section in Fig. 3

α_1	Lower limit value of α at which yielding in tension zones of the web and in the tension flange starts to penetrate
σ_{rc}, σ_{rt}	Maximum compressive and tensile residual stress, respectively
σ_y	Yield stress level
ϕ	Curvature
ϕ_y	Curvature corresponding to first yield in flexure

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